Mensionization Complementation

The Mathematics of Hermetic Alchemy

Section 5

Section 4 Continued, *Data Elements*, & the *Binomial Theorem*

In most cases, it is well known the use of series and polynomials, where applicable, can make math much simpler and more easily understood. Therefore, the *Duality* of *Form* Polynomial will be used to derive and explain the *transformations* and *expansions* within an α, β oppositional duality and along the way show another derivation and use of the coefficients in *Pascal's Triangle*. The *Duality* of *Form* polynomial is a mathematical summation equation that is a corner stone of *Men-Comp*. The polynomial's application not only applies to different disciplines of philosophy, it also includes many other traditions that use complementary α, β type oppositional dualities.

Since we are using *binomials* as equations for the opposing properties of dualities, the *Duality* of *Form* polynomial along with its differential and integral operators and the individual *Hermetic* volatile (α) and fixed (β) oppositional presences being the terms in the polynomial we will obtain a binomial *Dimensional Expansion Polynomial*, $E(\alpha+\beta)^n$. Its *terms* and *elements* can be easily derived by using *modules* from The *Duality* of *Form*'s summation equation**.**

11. Setting up a Dimensional Expansion Polynomial

$$\mathrm{E}(\alpha+\beta)^n = \sum_{k=0}^{n}\left\{\left(f^{k}(\alpha^{n})\right) \cdot \left(\int_{k} \frac{\beta^{0}}{0!}\right)\right\}$$

***(Working Form of the Duality of Form summation polynomial)* Eq.2**

$$E(\alpha+\beta)^n = f^0(\alpha)^n \cdot \int_0 (\frac{\beta^0}{0!})\, d\beta + f^1(\alpha^n)\cdot \int (\frac{\beta^0}{0!})\, d\beta + f^2(\alpha^n)\cdot \iint (\frac{\beta^0}{0!})\, d\beta + \cdots + \to n$$

Dimensional *Expanded* Polynomial) ***Eq.3***

Keep in mind, in the term ($k=0$), $\left(f^0(\alpha)^n \cdot \int_0 (\frac{\beta^0}{0!})\, d\beta \right)$ *no derivative or integration is performed which results in the term* ($k=0$)*, becoming the polynomial's constant term.*

When using the expanded *Duality* of *Form* equation as a *dimensional* expansion polynomial*;* it *correctly* reproduces the exact values of the coefficients of a binomial expansion as seen in *Pascal's* triangle. From these *permutations* or *coefficients*, we can determine the Binary (0, 1) *data-bit* element presences of the different oppositional *structures*. The binary structures will consist of different sub-codon *symbolic* configurations, each containing (α) solid lines ( ) and (β) broken lines ( ), representing the complementary changing inherent *volatile* (α) and *fixed* (β) oppositional element's presences.

Observe the *structure* of each individual term in the expanded polynomials shown above, **(**Equations 2 & 3**)**. In each term, we are multiplying successive differentiation of the *Generative Power* (α^n) volatile property (*contraction*), *times* the successive integration (*expansion*) of the *Productive Capacity's* $\frac{\beta^0}{0!}$ virtual fixed property. Each term in the expansion has the *same* pattern in which it produces easy to use " *modules*" as terms in the resulting polynomials. The polynomial Equation (3) above is exclusive for "*n*-dimensions", however, for this section we will only be working with *small-integer* number dimensions or oppositions on the order of *one* (1) to *three* (3), therefore we will only need to use a select few number of the modules ($n+1$) in the polynomial at any one time. As an example, to find the number of elements in a 2-dimensional system, we will only use three (3) modules ($n+1$) of the summation equation above to expand it, ($n+1$) modules for) $(n=2)$ dimensions or oppositions; and for a 3-dimensional system $(n=3)$ we'll only use four modules; also, ($n+1$) of the summation modules. We can better understand the expansion by observing the *Legend* of the simplified modular *Duality* of *Form* summation equation*,* (**Eq. 4** below).

11a. Duality of Form Summation Legend of the Modules

$$\mathrm{E}(\alpha+\beta)^{n}=\sum_{k=0}^{n}\left\{f^{k}(\alpha^{n})\ \cdot\ \int_{k}\left(\frac{\beta^{0}}{0!}\right)d\beta\right\}\quad \textbf{Eq. (4)}.$$

(The Simplified *working form* of the *Duality* of *Form* summation module equation)

Legend:

$E(\alpha+\beta)^{n}$ = *Expansion of an* α,β *oppositional or dimensional binomial.*

(α^{n}) = *The volatile presence of the opposition, a contracting masculine Generative Power.*

(β^{0}) = *The virtual-fixed presence of the opposition, an expanding feminine Productive Capacity.*

"n" = The number of dimensions or oppositions.

"k" used in, $f^{k}(\alpha^{n})$ & $\int_{k}$ *, is an integer index value of the summation process* $\left\{\sum_{k=0}^{n}(\ )\right\}$, *where* $k=0,\ 1,\ 2,\ +\cdots+,\ n$ *; the integer index value number of the successive derivatives and integration to perform on the* (α) *volatile and* (β) *fixed presences in each module. When* $(k=0)$*, no derivatives or integrals are performed and it becomes the 0-derivative, 0-integral constant term of the polynomial.*

In the following topics, by use of The *Duality* of *Form* summation polynomial we will build the *expanded* polynomials for *Hermetic Alchemy* (0-3 oppositions) and the *Lattice Datum's* 3-dimensions from the 0-dimension through the 3rd dimension and determine their presences plus show their data-elements *symbolically*.

12. Elements of the Zero-Dimensional Expansion – The Mension $f(m_{0})=(\alpha+\beta)^{0}$

Earlier in *Section 3*, the 0-dimension was defined by the graphic of the Eastern *T'ai Chi* in which some Alchemists & Eastern philosophers called a *Prima Materia* or virtual *unit*-quanta of *fuel* for creation. It was also expressed as a fundamental *virtual potential* opposition; suggesting it contains within itself a potential set of virtual (α^{0}), (β^{0}) opposites as its presences or elements.

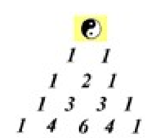
The *Mension* is a 0-dimensional unit and the *virtual* form of a fundamental 0-dimension of space or *alchemical* opposition.

The T'ai Chi

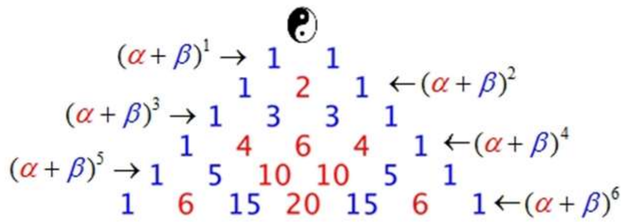
Therefore, for the *Mension's* illustrated diagram we will use the *T'ai Chi* as its *element* & *symbol*. Expanding this virtual dimension, we obtain:

$$E_{0}(\alpha+\beta)^{0}=(\alpha+\beta)^{0}=1.$$

$(\alpha+\beta)^{0}=1$ is a virtual *unit-bound* value with two (2) elements and is not explicitly expanded mathematically, although it *can* be. It is generally referenced similarly to the *T'ai Chi* as being just a *single* unit containing within itself Virtual α^{0} and β^{0} elements; a *composite* unit.



The resulting binomial *coefficients* in the expansions determine the number of *permutations* and type of binary (0, 1) bold line (α) ( ) and broken line (β) ( ) sub codon data elements, which also can be defined by the use of *Pascal's* triangle to more easily find these *coefficients* in each expansion. The graphic on the upper right is *Pascal's* triangle set up for the dimensional expansion of oppositions from the 0-dimension through the 4th-dimension. The 0-dimension position at the top, $(\alpha+\beta)^{0}$, has the *T'ai Chi's* symbol *highlighted* in place of a scalar 1 which normally occupies the top position. The use of the *T'ai Chi* in this position indicates it is being shown as a virtual *composite* unit and not explicitly as an element's *permutation* or *coefficient*.



Pascal's triangle is presently used in math operations to more easily find the higher *permutations* of *coefficients* involved in classic binomial expansions, however, we will instead use the expansion polynomial $E(\alpha+\beta)^n$ to expand the oppositional binomials to show another process to *derive* the exact coefficients of *Pascal's* triangle, and since *Pascal's* triangle is the expansion of binomials; the *expansion polynomial's* use of successive derivatives and integration is also an expansion of *Base* 2, α, β binomial oppositionals, which produce the exact same permutational *coefficient* element values as *Pascal's* triangle.

13. The Elements of the 1-Dimensional Expansion $f(m_1)=(\alpha+\beta)^1$

Performing an *Integration* on a composite function like the Mension $(\alpha+\beta)^0$, where (α) is the *independent* variable and has a coefficient and exponent of *one* (1), the result is simply an integration of the outer function divided by the exponent increase. We then obtain a *corporeal* 1-dimensional function.

$$f(m_1)=\int f(\alpha+\beta)^0 \, d\beta = \frac{(\alpha+\beta)^{0+1}}{1\cdot(0+1)} = (\alpha+\beta)^1$$

With Symbolic data Elements = α ***( )*** or β ***( )***

However, to expand a 1-dimension of space function when using the dimensional expansion polynomial $E(\alpha+\beta)^n$ we derive it using $(n+1)$ modules of the *Duality* of *Form* summation equation; we begin with an index of $(k=0)$, and $(n=1)$ equal to the number of dimensions to be expanded.

$$\mathrm{E}(\alpha+\beta)^1 = \sum_{k=0}^{1}\left\{ f^k(\alpha^1) \cdot \int_k \left(\frac{\beta^0}{0!}\right) d\beta \right\}$$

1-dimensional Expansion Summation Equation

We will be using only two (2) modules, $(n+1)=(1+1)$, of the above summation equation; a 0-dimensional *module* and a 1-dimensional *module* ($k=0$ & $k=1$). Then, replacing the value of the "n" exponent of the two $f^{k}(\alpha^{n})$ derivative terms with the number one (1), we will have an expanded polynomial for the expansion of a 1-dimensional function. k's Integer *value*-range is $k=(0,1)$.

$$E(\alpha+\beta)^{1}=\left(f^{0}(\alpha^{1})\cdot\int_{0}\frac{\beta^{0}}{0!}\,d\beta\right)+\left(f^{1}(\alpha^{1})\cdot\int_{1}\frac{\beta^{0}}{0!}\,d\beta\right)$$

(k=0), module/term 1 *(k=1), module/term 2*

In the first module/term $(k=0)$, we obtain a 0-*derivative*/0-*integral* first term, and the second module/term, is a 1st-*derivative*/1st *integral* term $(k=1)$. In the 0-*derivative*-0-*Integral* term $(k=0)$, no derivatives or integrals are performed, and it is equivalent to the polynomial *constant* term. Then simplifying the *modular* polynomials above we obtain:

$$E(\alpha+\beta)^{1}=f^{0}(\alpha^{1})\cdot(\beta^{0})+f^{1}(\alpha^{1})\cdot\int\left(\frac{\beta^{0}}{(0!)}\right)d\beta$$

(1-dimensional Expansion Polynomial)

Solving the above polynomial's *two* terms *individually* we obtain:

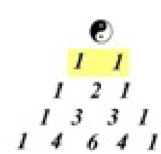
$$E_{0}(\alpha+\beta)^{1}=f^{0}(\alpha^{1})\cdot(\beta^{0})=(\alpha^{1})\cdot(\beta^{0})=1(\alpha^{1})$$

[k=0]-[1st term is a 0-derivative/0-integral Module].

$$E_{1}(\alpha+\beta)^{1}=f^{1}(\alpha^{1})\cdot\int\left(\frac{\beta^{0}}{0!}\right)d\beta=1(\alpha^{0})\cdot\frac{\beta^{(0+1)}}{(0!)\cdot 1}=1(\alpha^{0})\cdot\frac{(\beta)^{1}}{(1!)}=1(\beta^{1})$$

[k=1]-[2nd Term (1st derivative/1st integral Module).

Addition or *combining* the two terms, we then obtain: $E(\alpha+\beta)^{1}=1(\alpha)^{1}+1(\beta)^{1}$ which equals the *duality* of an (α) element and a (β) element.



The coefficients of the expansion of $f(\alpha+\beta)^{1}=1(\alpha)^{1}+1(\beta)^{1}$, *or (1, 1) is shown in* Pascal's Triangle (*right*) for a 1-dimensional oppositional binomial or a 1-dimension of space expanded function.

The dimensional expansion polynomial result, $\left(E(\alpha+\beta)^1 = 1\alpha \; + \; 1\beta\right)$ confirms the 1-dimensional system's data element is a *volatile* and a *fixed* sub codon element (▬) (α) and (β) (▬ ▬) of an opposition. Using a set of opposite *properties* such as "*wet*-*dry*" we would obtain a 1-dimensional binary line on the order of:

14. The 2-dimensional Complemented **Expansion.** $f(m_2) = (\alpha+\beta)^2$.

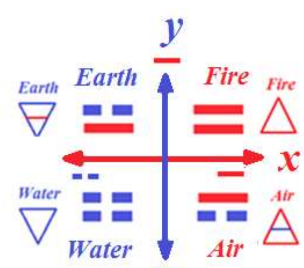
Multiplication of two (2), 1-dimension of space *binary* lines will result in a cross-multiplied 2-dimensional *orthogonally complemented* function.

$$f(\alpha+\beta)^2_{(x,y)} = (\alpha+\beta)^1_x \times \; (\alpha+\beta)^1_y$$

The 2-dimensional function is the first of the *orthogonally* complemented binomials. This expansion will be obtained in two (2) different ways; one by the *Dimensional Expansion Polynomial* and another, *algebraically*, by the above cross-multiplication product equation. Shown *left* below is the expanded equation. Also shown below *right* is its illustrated diagram. The *Four Codon Elements* of this expansion are shown as the *Four Elements* of the *Hermetic Alchemists* which will be explained further in later text.

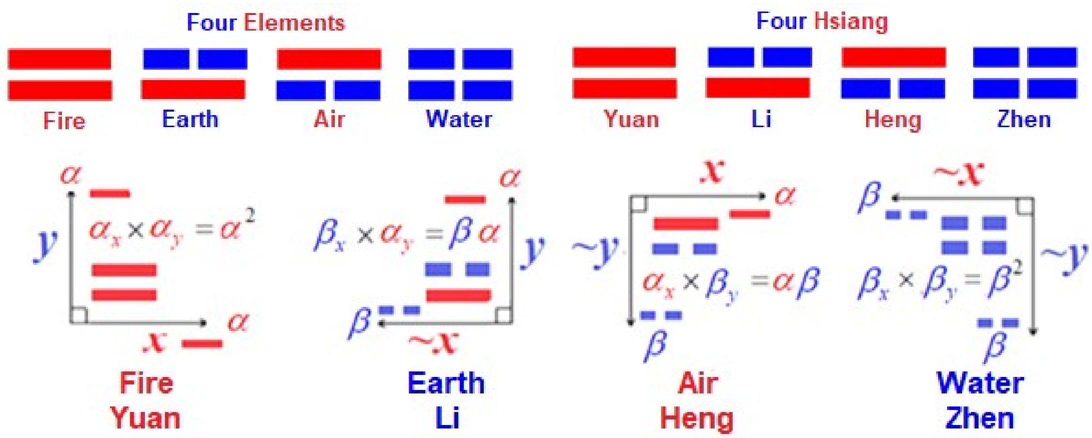
$$E(\alpha+\beta)^2_{(x,y)} = (\alpha+\beta)^1_x \times (\alpha+\beta)^1_y$$

$$E(\alpha+\beta)^2 = 1(\alpha)^2 + 2(\alpha^1 \times \beta^1) + 1(\beta)^2$$



Completing the *algebraic expansion* of the 2-dimensional model above, the cross-multiplication *complementary* product produces *four* (4) separate *areas*, *codons*, or *presences* showing the *Four Elements* of the 2-dimensional function, (2^2). Each of the *four area*-type-codons is a specific presence (or field) within the model above that defines the α, β of two complemented binary (***x***, ***y***) lines' presences. The *Four Elements* of the algebraically expanded equation and the *syncretic Four Hsiang* of the *I-Ching* are shown below in symbolic Binary (0, 1) (α)

( ) or (β) ( ), data "bit" *codon* notation.



4-element, 2-dimensional Bigram Binary Expanded Sub Codon Area Elements

Although the effects are different, the elements $(\beta_x \times \alpha_y)$ and the $(\alpha_x \times \beta_y)$, are algebraically equivalent by virtue of the mathematics being an *area* computation and *independent* of the unique *natures* or *qualities* of the *presences*. It is four *permutations* of one ( ) and ( ) *binary* combination. Each *codon* graphic in the 2-dimensional expansion is termed a "*Bigram*."

Expanding the 2-dimensional function by an expansion polynomial, only *three* (3) *modules* or *terms* of the *Duality* of *Form* polynomial are used. Once again, when using the *Duality* of *Form* polynomial for the expansions, use $(n+1)$ modules from the summation equation where "*n*" is the number of dimensions or oppositions to be expanded, e.g., expanding the 2-dimensional function, $(n=2)$, use # terms=$(n+1)$ or three (3) modules. Next, change the value of the exponent of "n" in each of the three $f^k(\alpha^n)$ successive derivative elements to the value of "$n=2$." When $n=2$ dimensions, we will need a 0-*derivative*/0-*integral* module $(k=0)$; a 1st-*derivative*/1st-*integral* module $(k=1)$, and a 2nd *derivative*/2nd *integral* module $(k=2)$. The summation equation is listed below for reference:

$$E(\alpha+\beta)^2 = \sum_{k=0}^{2}\left(f^k(\alpha^2)\cdot \int_k \left(\frac{\beta^0}{0!}\right) d\beta \right)$$

We obtain the following $(n+1)$ modules/terms:

$$E(\alpha+\beta)^2 = \left(f^0(\alpha^2)\cdot\int_0 \left(\frac{\beta^0}{0!}\right) d\beta \right) + \left(f^1(\alpha^2)\cdot\int_1 \left(\frac{\beta^0}{0!}\right) d\beta \right) + \left(f^2(\alpha^2)\cdot\int_2 \left(\frac{\beta^0}{0!}\right) d\beta \right)$$

$(k=0)$ *Term 1 Module* *(k=1) Term 2 Module* *(k=2) Term 3 Module*

Then simplify: where;

$$\int_0 \left(\frac{\beta^0}{0!}\right) d\beta = (\beta^0) \to \int_1 \left(\frac{\beta^0}{0!}\right) d\beta = \int \left(\frac{\beta^0}{0!}\right) d\beta \to \int_2 \left(\frac{\beta^0}{0!}\right) d\beta = \iint \left(\frac{\beta^0}{0!}\right) d\beta$$

$$n=2;\ k=0,1,2;\ \ (n+1)=3$$

We then obtain the 2-dimensional *expansion* polynomial.

$$E(\alpha+\beta)^2 = f^0(\alpha^2)\cdot(\beta)^0 + f^1(\alpha^2)\cdot\int\left(\frac{\beta^0}{0!}\right)d\beta + f^2(\alpha^2)\cdot\iint\left(\frac{\beta^0}{0!}\right)d\beta$$

$(k=0)$ *Term 1 Module* *(k=1) Term 2 Module* *(k=2) Term 3 Module*

Solving each of the *three* terms individually, we obtain the 4-permutation codons (1,2,1) shown in *Pascal's* triangle illustrated below right.

$$(k=0)\quad E_0(\alpha+\beta)^2 = f^0(\alpha^2)\cdot(\beta^0) = 1(\alpha^2)\cdot(\beta)^0 = 1(\alpha^2)$$

[*1st term, 0-derivative / 0-integral term module*]

$$(k=1)\quad E_1(\alpha+\beta)^2 = f^1(\alpha^2)\cdot\int\left(\frac{\beta^0}{0!}\right)d\beta = 2\left\{(\alpha^1)\cdot\frac{\beta^{(0+1)}}{(0!)\cdot 1}\right\} = 2\left\{(\alpha^1)\cdot\frac{(\beta)^1}{(1!)}\right\} = 2(\alpha^1\times\beta^1)$$

[*2nd term, 1st derivative / 1st integral module*]

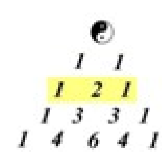
$$(k=2)\quad E_2(\alpha+\beta)^2 = f^2(\alpha^2)\cdot\int\frac{\beta^1}{(1!)}\,d\beta = 2(\alpha^0)\cdot\frac{(\beta)^{(1+1)}}{(1!)\cdot 2} = \cancel{2}(\alpha^0)\cdot\frac{(\beta)^2}{\cancel{2!}} = 1(\alpha^0\cdot\beta^2) = 1(\beta^2)$$

[*3rd term, 2nd derivative / 2nd integral] module*]

Giving the following result:

$$E(\alpha+\beta)^2 = 1\big((\alpha^2)\cdot(\beta^0)\big) + 2\big((\alpha^1)\cdot(\beta^1)\big) + 1\big((\alpha^0)\cdot(\beta^2)\big)$$

Again, in the *expansion* of the 2-dimensional binomial by the *Dimensional Expansion Polynomial*, we obtain the exact same permutation coefficients or elements (1, 2, 1) shown highlighted in *Pascal's* triangle on the right.



5. The Lattice Datum – 3-dimensional Orthogonal Complementation

$$f(m_3) = (\alpha + \beta)^3$$

This complementary expansion will also be obtained two ways, one by the dimensional expansion polynomial and another *algebraically* by the following cross-multiplication product.

$$(\alpha + \beta)^3_{(x,z,y)} = (\alpha + \beta)^1_x \times (\alpha + \beta)^1_z \times (\alpha + \beta)^1_y$$

$$(\alpha + \beta)^3 = 1\left[(\alpha^3) \times (\beta^0)\right] + 3\left[(\alpha^2)\cdot(\beta^1)\right] + 3\left[(\alpha^1)\cdot(\beta^2)\right] + 1\left[(\alpha^0)\cdot(\beta^3)\right].$$

Expanding the 3-dimensional *Lattice Datum* system by the *Duality* of *Form* Polynomial, use four (4) modules ($Terms = n + 1 = 4$) and changing each "n" exponent of the $f^k(\alpha^n)$ derivative components to (3), we obtain.

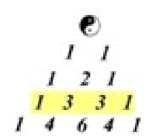
$$E(\alpha + \beta)^3 = \left(f^0(\alpha^3)\cdot\int_0 \frac{(\beta^0)}{0!}\, d\beta \right) + \left(f^1(\alpha^3)\cdot\int_1 \frac{(\beta^0)}{0!}\, d\beta \right) + \left(f^2(\alpha^3)\cdot\int_2 \frac{(\beta^0)}{0!}\, d\beta \right) + \left(f^3(\alpha^3)\cdot\int_3 \frac{(\beta^0)}{0!}\, d\beta \right)$$

(k=0) Module 1 *(k=1) Module 2* *(k=2) Module 3* *(k=3) Module 4*

Again simplifying where $n = 3,\ k = 0, 1, 2, 3$ we obtain the 3-dimensional expansion polynomial:

$$E(\alpha + \beta)^3 = f^0(\alpha^3)\cdot(\beta^0) + f^1(\alpha^3)\cdot\int \frac{\beta^0}{0!}\, d\beta + f^2(\alpha^3)\cdot\iint \frac{\beta^0}{0!}\, d\beta + f^3(\alpha^3)\cdot\iiint \frac{\beta^0}{0!}\, d\beta$$

Solving each module-term individually for simplicity we obtain the eight 1^{st} order volume elements shown in *Pascal's* triangle (below right. (1-3-3-1).



$$f^0(\alpha^3)\cdot(\beta^0) = 1(\alpha^3)$$

$(k = 0)$ *Term [0-derivative-0-integral term]*

$$f^1(\alpha^3)\cdot\int\left(\frac{\beta^0}{0!}\right) d\beta = 3(\alpha^2)\cdot\frac{\beta^{(0+1)}}{(0!)\cdot 1} = 3\left((\alpha^2)\cdot\frac{\beta^1}{(1!)}\right) = 3\alpha^2\cdot\beta^1;$$

$(k = 1)$ *Term* $[1^{st}$ *derivative,* 1^{st} *integral*$]$

$$f^2(\alpha)^3\cdot\iint\frac{\beta^0}{0!}\, d\beta = 6(\alpha^1)\cdot\int\frac{\beta^1}{(1!)} = 6(\alpha^1)\cdot\frac{(\beta)^{(1+1)}}{(1!)\cdot 2} = \cancel{6}\left((\alpha)^1\cdot\frac{(\beta)^2}{(\cancel{2}!)}\right) = 3\alpha^1\cdot\beta^2;$$

$(k = 2)$ *Term* $[2^{nd}$ *derivative,* 2^{nd} *integral*$]$

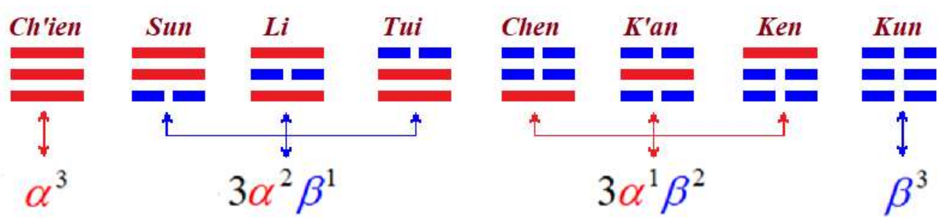
$$f^3(\alpha^3)\cdot\iiint\frac{\beta^0}{(0!)}\,d\beta=6(\alpha^0)\cdot\int\frac{\beta^2}{(2!)}\,d\beta=\cancel{6}(\alpha^0)\cdot\frac{1}{(2!)}\cdot\frac{\beta^{(2+1)}}{(2+1)}=6(\alpha^0)\cdot\frac{\beta^3}{(2!)\cdot 3}=\cancel{6}(\alpha^0)\cdot\frac{\beta^3}{\cancel{(3!)}}=1(\beta^3)$$

$(k=3)$ *Tern* [*3rd Derivative, 3rd integral*]

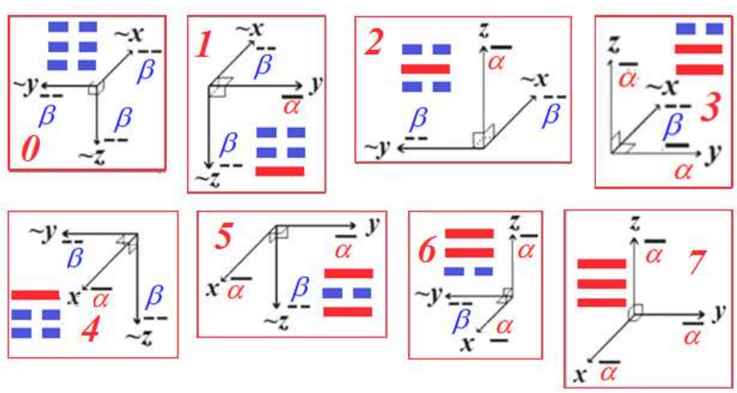
Which again gives coefficients of $1(\alpha^3)+3\left[(\alpha^2)\cdot(\beta^1)\right]+3\left[(\alpha^1)\cdot(\beta^2)\right]+1(\beta^3)$ or 1-3-3-1, the *eight* (8) elemental volume presences as shown in *Pascal's* triangle highlighted on the above right.

The two (2) Pascal *coefficients* of value *three* (3) are the *three* (3) *permutations* of the *trigram* terms $3\alpha^2\beta^1$ and $3\alpha^1\beta^2$ shown below in the expanded graphic which resulted from the derivative/integral process.

The Mathematical 3-dimensional Eight Trigram Volume Elements



The next graphic contains all the above including the up-coming *Binary Value trigrams*.



The *red* numbers in the above graphic are the *binary value* numbers of the different eight (8) *trigrams* (also termed as "*octants*") shown above, where the top line of the *trigram* or *octant* represents the ***x*** axis, the middle line the ***zed*** axis, and the bottom line or *line* (1) the ***y*** axis. The specific ordering of the ***x***, then ***zed***, & then ***y*** will be explained in an upcoming section on the 2*nd* & 3*rd* *Dimensional* systems.

You may have noticed I called the *eight* trigrams "1st *order-elements*." The true graphic illustration (or "***Engine***") of the 3-dimensional *Lattice Datum* system is actually a 6-figured $(\alpha+\beta)^1$ graphic figure called a "*Hexagram*." It is a result from a 2nd derivative of the binomial $f(m_3)=(\alpha+\beta)^3$; where the *Hexagram* is termed a *first differential* state, $(\alpha+\beta)^1$ thus enclosed in solid *red parentheses* and *exponent*. It will be explained in *Section* 6, along with some inner operational nuances of the interior math in *Hermetic Alchemy*.

Lateral 4

Rearranging Equations in the *Binomial Theorem*

$$E(\alpha+\beta)^n=\sum_{k=0}^{n}\left(\frac{d^k}{d\alpha^n}(\alpha^n)\cdot\int_k\left(\frac{\beta^0}{0!}\right)d\beta\right)$$

The Duality of Form Summation Equation

Recall from *Section* 4, the power form polynomial below is equivalent to $f(e^\beta)$:

$$f(e^\beta)=\sum_{k=0}^{n}\left(\frac{(\beta^k)}{(k!)}\right)=\left\{\frac{\beta^0}{0!}+\frac{\beta^1}{1!}+\frac{\beta^2}{2!}+\frac{\beta^3}{3!}+\frac{\beta^4}{4!}+\frac{\beta^5}{5!}+\cdots+\frac{\beta^n}{n!}\right\}$$

If we substitute an equivalent form of the last term $\frac{(\beta^n)}{(n)!}$ in the above power form polynomial into the original *Duality* of *Form* polynomial equation shown at the beginning of this topic, replacing the *integration* term; the resulting outcome will be the first binomial polynomial form, which will be much easier to work with.

$$E(\alpha+\beta)^n=\sum_{k=0}^{n}\left(\frac{d^k}{d\alpha^n}(\alpha)^n\cdot\frac{(\beta)^k}{(k)!}\right)$$

The Adjusted Duality of Form Polynomial.

We will now do some more *rearranging* of the classic binomial equations. In the following equations below, notice the *similarity* of the above equation to the binomial equation that is used for *combinations*. When we alter the classic binomials, the *blue* $(k!)$ factorial of the equation below, with a small *adjustment*, is equivalent to the number of *permutations* of (α^{n-k}).

$$ {}_nC_k = \frac{n!}{k!\cdot(n-k)!}\cdot\alpha^{(n-k)}\cdot\beta^k $$ [Factorial Binomial for Combinations]

If we *move* the singular blue ($k!$) under the β^k term, as a *divisor* and change the equation to a *summation* equation, we get a similar equation to the Duality of Form equation where the *coefficient* of $\alpha^{(n-k)}$ is equal to the *formula* for *permutations*.

$$ f(\alpha+\beta)^n = \sum_{k=0}^{n}\frac{n!}{(n-k)!}\cdot\alpha^{(n-k)}\cdot\frac{\beta^k}{(k!)} $$

(The Factorial version of the Duality of Form equation)

- The equation above gives us a *factorial* version of the *Duality* of *Form* equation and the beginning term instead of being *successive derivatives* as in the *Duality* of *Form* equation, now completely eliminates the use of the *successive derivatives* and *successive integration*. Since the coefficients of derivatives have been proven to be *permutations*, the *Duality* of *Form* polynomial equation and the one shown above are *equivalent* equations; this equivalency was in part used in the proof in *Section 4* showing *permutations* are equivalent to the *coefficients* of derivatives.

$$ \sum_{k=0}^{n}\left\{\frac{n!}{(n-k)!}\cdot\alpha^{(n-k)}\right\} = \sum_{k=0}^{n}\left\{\frac{d^k}{d\alpha^n}\alpha^n\right\} $$

We have already shown the proof earlier in the text of *Section* 4, so we will not repeat it again.

The above factorial transformed equation has been renamed the *Factorial Duality* of *Form* equation from whence it originated. It is also equivalent to the expansion of an α,β oppositional binomial. In other words; the following two summation equations shown below are equivalent, they produce the exact same output (*Pascal's* triangle and oppositional α,β solutions).

$$ \sum_{k=0}^{n}\left(\frac{d^k}{d\alpha^n}(\alpha)^n\cdot\int_k\left(\frac{\beta^0}{0!}\right)d\beta\right) = \sum_{k=0}^{n}\left\{\frac{n!}{(n-k)!}\cdot(\alpha)^{(n-k)}\cdot\frac{\beta^k}{(k!)}\right\} $$

The Duality of Form Summation = The Factorial Duality of Form Summation

We have already seen the expansion of the *left* side of the above equality previously in this section and have shown it gives us the *coefficients* of *Pascal's* triangle; now the *right* side of the equality above will be expanded to show that it is the *equivalent* of the *left* side, they both are equivalent to the *coefficients* of *Pascal's* triangle, plus, solutions to α, β oppositionals.

Lateral 5

The Expansion of the Factorial Duality of Form Equation

$$\mathrm{E}(\alpha+\beta)^n = \sum_{k=0}^{n}\left\{\frac{n!}{(n-k)!}(\alpha)^{(n-k)}\cdot\frac{\beta^k}{(k!)}\right\}$$

The 0-dimensional Virtual System $(n=0)$ *Tai' Chi*

$$(k=0)\quad \left(\left[\frac{n!}{(n-k)!}\right](\alpha^{n-k})\cdot\frac{(\beta)^n}{n!}\right)=\left(\left[\frac{0!}{(0-0)!}\right](\alpha^{(0-0)})\cdot\frac{\beta^0}{0!}\right)=\frac{0!}{0!}(\alpha^0)\cdot\frac{\beta^0}{0!}=\left[(\alpha^0)\cdot(\beta^0)\right]$$

The 1-dimensional Oppositional System $(n=1),(k=0,1)$

$$(k=0)\quad \left(\left[\frac{n!}{(n-k)!}\right](\alpha^{n-k})\cdot\frac{(\beta)^n}{n!}\right)=\left[\frac{1!}{(1-0)!}\right](\alpha^{(1-0)})\cdot\frac{\beta^0}{0!}=\frac{1!}{1!}(\alpha^1)\cdot\frac{(\beta)^0}{0!}=1(\alpha)^1$$

$$(k=1)\quad \left(\left[\frac{n!}{(n-k)!}\right](\alpha^{n-k})\cdot\frac{(\beta)^n}{n!}\right)=\left[\frac{1!}{(1-1)!}\right](\alpha^{(1-1)})\cdot\frac{\beta^1}{1!}=\frac{1!}{0!}(\alpha^0)\cdot\frac{\beta^1}{1!}=1(\beta^1)$$

The 2-dimensional Oppositional System $(n=2)$ **(*Four Elements*)** $(k=0,1,2)$

$$(k=0)\quad \left(\left[\frac{n!}{(n-k)!}\right](\alpha^{n-k})\cdot\frac{(\beta)^n}{n!}\right)=\left[\frac{2!}{(2-0)!}\right](\alpha)^{(2-0)})=\frac{2!}{2!}(\alpha)^2\cdot\frac{\beta^0}{0!}=1(\alpha^2)$$

$$(k=1)\quad \left(\left[\frac{n!}{(n-k)!}\right](\alpha^{n-k})\cdot\frac{(\beta)^n}{n!}\right)=\left[\frac{2!}{(2-1)!}\right]\alpha^{2-1}=\frac{2!}{1!}\alpha^{2-1}=2(\alpha^1)\cdot\frac{\beta^1}{1!}=2\alpha^1\cdot\beta^1$$

$$(k=2)\quad \left(\left[\frac{n!}{(n-k)!}\right](\alpha^{n-k})\cdot\frac{(\beta)^n}{n!}\right)=\left[\frac{2!}{(2-2)!}\right](\alpha^{2-2})\cdot\frac{\beta^2}{2!}=\frac{2!}{0!}(\alpha^0)=\frac{2!}{0!}(\alpha^0)\cdot\frac{\beta^2}{2!}=1(\beta^2)$$

The 3-dimensional Lattice Datum $(n=3),(k=0,1,2,3)$

$$(k=0)\quad \left(\left[\frac{n!}{(n-k)!}\right](\alpha^{n-k})\cdot\frac{(\beta)^n}{n!}\right)=\frac{3!}{(3-0)!}(\alpha^{3-0})\cdot\frac{\beta^0}{0!}=\frac{\cancel{3!}}{\cancel{3!}}(\alpha^3)\cdot\frac{\beta^0}{0!}=(\alpha^3\cdot\beta^0)=1(\alpha^3)$$

$$(k=1)\quad \left(\left[\frac{n!}{(n-k)!}\right](\alpha^{n-k})\cdot\frac{(\beta)^n}{n!}\right)=\frac{3!}{(3-1)!}(\alpha^{3-1})\cdot\frac{\beta^1}{1!}=\frac{(\cancel{2!}\cdot 3)}{(\cancel{2!})}=3(\alpha^2)\cdot\frac{\beta^1}{1!}=3\alpha^2\cdot\beta^1$$

$$(k=2)\quad \left(\left[\frac{n!}{(n-k)!}\right](\alpha^{n-k})\cdot\frac{(\beta)^n}{n!}\right)=\frac{3!}{(3-2)!}(\alpha^{3-2})\cdot\frac{\beta^2}{2!}=\frac{3!}{1}(\alpha^1)\cdot\frac{\beta^2}{2!}=\cancel{6}(\alpha^1)\cdot\frac{\beta^2}{\cancel{2!}}=3\alpha^1\cdot\beta^2$$

$$(k=3)\quad \left(\left[\frac{n!}{(n-k)!}\right](\alpha^{n-k})\cdot\frac{(\beta)^n}{n!}\right)=\frac{3!}{(3-3)!}(\alpha^{3-3})\cdot\frac{\beta^3}{3!}=\frac{3!}{0!}(\alpha^0)\cdot\frac{\beta^3}{3!}=\cancel{3!}(\alpha^0)\cdot\frac{\beta^3}{\cancel{3!}}=1(\beta^3)$$

Using the *Factorial* Equation, we output the same values for *Pascal's* triangle, as did with the other two binomial equations. The advantage we have using *binomials* as oppositional functions is that we have all the equations of the *Binomial Theorem* available, which contains a vast amount of *analytical* mathematics.

Take note that since the binomial *Duality* of *Form* polynomial is just another route to solving binomials, any of the numerous other *classic* binomial formulas/equations should be valid as well if properly *predicated* and some will noticeably be simpler to work with. However, the *Duality* of *Form* polynomial has a slight advantage by describing the internal mathematics of α,β oppositional systems and its use within other *philosophical disciplines*.

We will conclude this section with some "*Thought Experiments*" on possible ways to use the *Duality* of *Form* equation to better understand how they may be used in the real world. In this section and the previous section you have been introduced to the algebraic math which will transform any set of *true* oppositions. The α and β are a *set* of *true* opposites; replace the α and β with meaningful terms that express each opposition and determine if it is correct or not. These are examples, try some of your own. The 3-dimensional hierarchy is given below.

$$\sum_{k=0}^{n}\left(\frac{d^k}{d\alpha^n}(Good)^n \cdot \int_k \frac{(Evil)^0}{0!})\,d\beta\right)$$

1st Dimension

$$(Good)^1 \cdot (Evil)^0 + (Good)^0 \cdot (Evil)^1$$

2nd Dimension

$$(Good)^2 \times (Evil)^0 + 2(Good)^1 \cdot (Evil)^1 + (Good)^0 \cdot (Evil)^2$$

3rd Dimension

$$(Good)^3 \cdot (Evil)^0 + 3(Good)^2 \cdot (Evil)^1 + 3(Good)^1 \cdot (Evil)^2 + (Good)^0 \cdot (Evil)^3$$

Resolve the Following thought Suggestions.

$$\sum_{k=0}^{n}\left(\frac{d^k}{d\alpha^n}(Hot)^n \cdot \int_k \frac{(Cold)^0}{0!})\,d\beta\right)$$

$$\sum_{k=0}^{n}\left(\frac{d^k}{d\alpha^n}(Time)^n \cdot \int_k \frac{(Space)^0}{0!})\,d\beta\right)$$

(*The Binary Number System Section 5-A*)